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Uncertainty in Phase Arrival Time Picks for Regional Seismic Events: An Experimental Design

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Abstract

The detection and timing of seismic arrivals play a critical role in the ability to locate seismic events, especially at low magnitude. Errors can occur with the determination of the timing of the arrivals, whether these errors are made by automated processing or by an analyst. One of the major obstacles encountered in properly estimating travel-time picking error is the lack of a clear and comprehensive discussion of all of the factors that influence phase picks. This report discusses possible factors that need to be modeled to properly study phase arrival time picking errors. We have developed a multivariate statistical model, experimental design, and analysis strategy that can be used in this study. We have embedded a general form of the International Data Center(IDC)/U.S. National Data Center(USNDC) phase pick measurement error model into our statistical model. We can use this statistical model to optimally calibrate a picking error model to regional data. A follow-on report will present the results of this analysis plan applied to an implementation of an experiment/data-gathering task.

1 Introduction

The detection and timing of arrivals of seismic waves play a critical role in our ability to locate seismic events, especially those of low magnitude. One of the major obstacles encountered in properly estimating travel-time picking error is the lack of a clear and comprehensive discussion of all of the sources of variance in measured travel times. This paper discusses possible sources that may impact arrival-timing errors and develops a statistical framework that can then be utilized for modeling picking error estimation. We begin with a review of the current methodology for dealing with measurement error for event location. In Section 2 we discuss factors that influence signal quality. Section 3 discusses analyst factors that influence phase arrival time picking error. In Section 4 we propose an experimental design and statistical model that can be used to study factors that influence phase arrival pick errors and calibrate a pick error model to regional data. In Section 5 we review the data analysis process associated with our statistical model. Section 6 summarizes the paper and describes a planned follow-on report.

1.1 Uncertainty Propagation for Event Location

In the location method currently employed by both the U.S. National Data Center(USNDC) and the International Data Centre(IDC), the weighting of arrival picks is formulated as the quadratic sum of modeling (σ^2_{Model}) and measurement ($\sigma^2_{Measurement}$) errors, where the total error for an arrival is $\sigma^2_{Total} = \sigma^2_{Model} + \sigma^2_{Measurement}$. The inverse of the total error assigned to the arrival time is the weight used in the location inversion. In current operations, the modeling error for each phase has been defined as a simple function of distance derived by comparing model predictions with measured arrival times for ground truth events. More recent work has focussed on extending the functional dependence to two or three dimensions with the use of various interpolation algorithms such as kriging [Schultz et al., 1998]. In fact, these are Bayesian methods that embed 2-D or 3-D empirically derived information where available into a simple radially distance-dependent background model, hence assuring the best results both for calibrated and uncalibrated regions. The treatment of measurement error is even simpler. At the IDC/USNDC, phase pick measurement error is currently modeled as a function of signal-to-noise ratio (SNR) only. The general model is

$$\sigma_{Measurement} = \begin{cases} \sigma_0 \text{ sec} & \text{if SNR} < \theta_L \\ \gamma\ \sigma_0 \text{ sec} & \text{if SNR} > \theta_U \qquad 0 < \gamma < 1. \\ \sigma_0 - \frac{\sigma_0 - \gamma\ \sigma_0}{log_{10}(\theta_U) - log_{10}(\theta_L)}\ log_{10}(SNR/\theta_L) \text{ sec} & \text{otherwise} \end{cases} \tag{1}$$

This simple model accounts for the most obvious and perhaps the most significant factor which contributes to the measurement error, namely the quality of the signal, but as we shall show in this paper, it ignores many other factors which, we believe, can and should be accounted for.

1.2 Regional vs. Teleseismic Phase Arrival Picks

The current measurement error estimate for teleseismic P-wave phase arrivals, which is based on the SNR alone, has the functional form given in Equation 1. This model must be properly fitted to regional data. Regional phases travel through the heterogeneous crust and upper mantle, which usually results in more than one raypath, and hence emergent arrivals even when the SNR is high. In contrast, teleseismic P-waves usually have just one raypath because they travel mostly through the mantle. Thus, by their nature, regional phases are usually more difficult to pick, either by an automated process or by an analyst, and therefore exhibit higher measurement errors for comparable SNR signals.

1.3 Factors Affecting Phase Arrival Timing

Two basic factors determine how accurately the travel time of a phase can be picked:

- the quality of the observed seismic phase and
- the technical training and experience of the analyst making the pick.

The signal quality impacts both automated detection and an analyst's ability to pick an arrival. We begin by discussing the signal quality effects and then the effects that impact analysts.

2 Signal Quality Factors

In this section, we briefly discuss the factors that affect either the signal or noise characteristics. Some of these factors can be accounted for directly with analytic formulas, whereas others are more appropriately dealt with as probability distributions. We group the factors into three sections: source effects, propagation/site effects, and other effects.

2.1 Source Factors

Several source effects can impact the quality of phase arrivals: magnitude, phase excitation, focal mechanism, and the source time function. For event magnitude (size), larger events usually lead to larger phase amplitudes (higher SNR), which should make the pick more accurate. Even emergent arrivals can be easier to pick if they exhibit higher SNR. Phase excitation, or the amount of energy coupled into each phase, is not expected to be the same for all phases. The same event might be expected to have smaller amplitudes (and therefore a higher picking error) for one phase vs. another. The focal mechanisms for earthquakes (double couple sources) dictate that energy does not radiate equally in all directions. Thus, phase amplitudes vary with take-off angle of the ray from source to receiver and the azimuth between source and receiver. Analytic formulas are available to account for the radiation patterns if the focal mechanism is known. The source time function can also impact the quality of observed seismic phases. Some seismic sources are impulsive; thus they have impulsive arrivals. Others build slowly, causing more emergent arrivals. Others still can rupture in unilateral motions, contributing to directivity effects, which can cause impulsive arrivals in some directions and emergent arrivals in others. Other source effects relate to frequency content, such as the corner frequency. Seismic sources radiate energy across a range of frequencies (i.e. a spectrum). More high-frequency energy could lead to more accurate picks, though this depends on the attenuation properties of the Earth, the band pass of the instrument, and any filtering applied by the analyst.

2.2 Propagation Path and Site Factors

Recordings of arrivals farther away from a source mean smaller amplitudes due to geometric spreading (not frequency dependent) and anelastic attenuation (frequency dependent). Thus, picks should be less accurate as distance increases. Both of these factors vary according to phase and to region. For example, S_n spreads differently and attenuates differently than P_n . Further, S_n for one region may spread differently and attenuate differently than S_n in another region, even for adjacent regions (e.g. western U.S. vs. eastern U.S.).

Near-source and near-receiver noise can also impact signal quality. Though not generally considered, noise near the source can degrade picking accuracy. For example, an earthquake occurring within the coda of another earthquake would be expected to have a larger error in picking than the same event without another event preceding it. Near-receiver effects are at least as important: if the background noise for a given station is lower, the pick will be more accurate.

2.3 Other Factors

Secondary phases have increased noise characteristics due to the fact that they are in the coda of other arrivals. Therefore, picking of secondary phases can be dependent on the characteristics of the phase that preceded it. The station type and instrument noise can also affect signal quality. For example, an array has a better signal quality than a single channel (the improvement is proportional to the square root of the number of array elements). However, this is only true if the design of the array is matched to the signal characteristics. For example, using a large-aperture array (teleseismic) for regional signals does not yield as much improvement as using a smaller aperture array (regional). Finally, more accurate arrival picks can be made from an instrument with lower noise characteristics.

3 Analyst Factors

Analyst factors are those characteristics of an analyst that we believe are likely to have an impact on travel-time picking, independent of signal quality. Though there are many potential influences on how an analyst makes picks, (e.g. some analysts may pick better in the morning than the afternoon) we discuss only the most significant.

3.1 Analyst-to-Analyst Variability

Given the same seismogram, different analysts pick the same phase differently. This would be true even if the analysts had the same level of experience and training (see below). Phase arrival time picks are dependent on the frequency band used in the pick. Analysts may have the discretion to use their "favorite" suite of frequency bands even when constrained by an analysis paradigm. We aggregate analyst-to-analyst variability into model error (the covariance matrix of the model presented in Section 4). We do not attempt to model individual analyst uncertainty. We claim that such a study is not feasible because the experiment would require multiple measurements, over time, from each analyst. This experiment would change what it is designed to study (analysts) — there would be a learning effect that could not be removed. This learning effect would result in a very poor estimate of individual analyst variability. If this learning effect could be removed, a well established, repeated measures design could be applied; however the learning effect is problematic.

3.2 Analyst Experience

If it were possible to present an analyst with the same waveform several times over some duration of time without the analyst recalling the waveform, for all but the highest SNR signals, it is almost certain that a particular analyst would pick the same phase differently. We would expect the analyst picks to be more consistent as the experience level of the analyst increases; i.e. the variance of the residuals should decrease, though the bias might not.

3.3 Analyst Training

Analyst training could be lumped together with analyst experience, but one can also make a case for separating the two. Simply increasing the experience should decrease residual variance. On the other hand, if two or more analysts are provided with the same training, this should have the effect of abating disagreement between their picks. Increased experience alone is not guaranteed to do this.

4 An Experimental Design and Statistical Model to Study Uncertainty in Phase Arrival Picks

Having discussed the various factors that can affect phase picking, we now propose a model for regional phase pick uncertainty that accounts for many of these factors. A multivariate data point is composed of the phase arrival time picks (adjusted with automated picks) from two analysts from the same seismic event. Without loss of generality we consider the P_n and S_n phases. Let T_{P_n} and T_{S_n} denote the automated phase arrival time picks for the P_n and S_n phases. Denote the analysts' picks as $Y_{P_{n1}}, Y_{P_{n2}}$ (P_n phase arrival time picks) and $Y_{S_{n1}}, Y_{S_{n2}}$ (S_n phase arrival time picks). The multivariate data point is denoted

$$\mathbf{Y} = \begin{pmatrix} Y_{P_{n1}} - T_{P_n} \\ Y_{S_{n1}} - T_{S_n} \\ Y_{P_{n2}} - T_{P_n} \\ Y_{S_{n2}} - T_{S_n} \end{pmatrix}. \tag{2}$$

Note that it is not essential that the automated phase picks be "truth" for our development. They simply provide a common objective reference point for the analyst picks. If ground truth were available for a set of picks, we could then compare both analysts' picks with ground truth and develop a model that would account for analyst bias as well as error. Based on our discussion of the factors affecting measurement error, we select as the dominant model factors that influence $\mathbf{Y}$:

- $\mathbf{S}_{i_1}$ — Event source (i.e., type) with nominal levels $i_1 = 1, 2, \ldots, s$
- $\mathbf{P}_{i_2}$ — Wave path or geophysical medium with nominal levels $i_2 = 1, 2, \ldots, p$
- $\mathbf{E}_{i_3}$ — Experience with nominal levels $i_3 = 1, 2, 3$
- $\mathbf{T}_{i_4}$ — Paradigm training with nominal levels $i_4 = 1, 2$
- $\mathbf{C}_{i_5}$ — Station configuration with nominal levels $i_5 = 1, 2, 3$
- $\mathbf{I}_{i_6}$ — Seismometer type with nominal levels $i_6 = 1, 2, 3$
- $mb_{i_1 i_2 i_3 i_4 i_5 i_6 j}$ — Event magnitude covariate (continuous regression variable)
- $\Delta_{i_1 i_2 i_3 i_4 i_5 i_6 j}$ — Epicentral distance covariate (continuous regression variable)
- $x_{i_1 i_2 i_3 i_4 i_5 i_6 j}$ — Signal-to-noise ratio (SNR) (continuous regression variable)
- $\boldsymbol{\epsilon}_{(i_1 i_2 i_3 i_4 i_5 i_6) j}$ — Measurement error and model inadequacy with levels $j = 1, 2, \ldots, n$.

When formally writing the model, we subscript the multivariate data vector as $\mathbf{Y}_{i_1 i_2 i_3 i_4 i_5 i_6 j}$. We dispense with the subscripts when there is no loss of clarity. The covariates mb, Δ, and, x

are regression variables with the usual assumption of being nonrandom. We define the weight function

$$w(x,\gamma) = \begin{cases} 1 & \text{if x} < \theta_L \\ \gamma & \text{if x} > \theta_U \\ 1 - \frac{1-\gamma}{log_{10}(\theta_U) - log_{10}(\theta_L)}\, log_{10}(x/\theta_L) \text{ sec} & \text{otherwise} \end{cases} \qquad 0 < \gamma < 1. \tag{3}$$

Here, θ_L and θ_U are assumed known and as discussed in Section 5, γ is estimated, along with other model parameters, with regional data. Note the similarity between Equations 1 and 3 — apart from the multiple σ_0 they are the same. Because phase pick measurement error is a function of SNR, we adjust (weight) the data vector $\mathbf{Y}$ and form a new vector

$$\mathbf{Z} = \frac{\mathbf{Y}}{w(x,\gamma)}. \tag{4}$$

Weighting $\mathbf{Y}$ to form the new vector $\mathbf{Z}$ is necessary to satisfy standard assumptions of constant covariance structure in multivariate analysis of variance (see [Rencher, 1995] and [Rencher, 1998]). We show, in the following paragraphs, that the weighting Equation 3 is central to the phase pick uncertainity model developed in this report. For the new data vector $\mathbf{Z}$, a full factorial multivariate analysis of covariance model (MANCOVA) is

$$\begin{aligned} \mathbf{Z}_{i_1\,i_2\,i_3\,i_4\,i_5\,i_6\,j} = {} & \text{Covariates} + \text{Main Effects} + \\ & \text{Two-way Interactions} + \text{Three-way Interactions} + \ldots + \\ & \text{Six-way Interaction} + \boldsymbol{\epsilon}_{(i_1\,i_2\,i_3\,i_4\,i_5\,i_6)\,j}. \end{aligned} \tag{5}$$

This model and associated analysis is unwieldy. We suggest a station-centric approach to reduce the dimensionality of the problem and thereby eliminate the higher-order interactions. This parsimonious approach allows a multivariate analysis on data from a specific station configuration, seismometer type, analyst experience and paradigm training, thereby allowing us to eliminate the factors $\mathbf{E}$,$\mathbf{T}$,$\mathbf{C}$ and $\mathbf{I}$ by holding them constant. The model for this approach is

$$\begin{aligned} \mathbf{Z}_{i_1\,i_2\,j} = {} & \boldsymbol{\mu} + \alpha\, mb_{i_1\,i_2\,j} + \beta\, \Delta_{i_1\,i_2\,j} + \mathbf{S}_{i_1} + \mathbf{P}_{i_2} + \mathbf{SP}_{i_1\,i_2} + \boldsymbol{\epsilon}_{(i_1\,i_2)\,j} \\ & i_1 = 1,2,\ldots,s;\; i_2 = 1,2,\ldots,p;\; j = 1,2,\ldots,n. \end{aligned} \tag{6}$$

We can further clarify Equation 6 by combining the $\mathbf{S}$, $\mathbf{P}$, and $\mathbf{SP}$ terms into a single mean term (see [Rencher, 1995] and [Rencher, 1998]). This "cell-means" representation is

$$\begin{aligned} \mathbf{Z}_{i_1\,i_2\,j} = {} & \boldsymbol{\mu}_{i_1\,i_2} + \alpha\, mb_{i_1\,i_2\,j} + \beta\, \Delta_{i_1\,i_2\,j} + \boldsymbol{\epsilon}_{(i_1\,i_2)\,j} \\ & i_1 = 1,2,\ldots,s;\; i_2 = 1,2,\ldots,p;\; j = 1,2,\ldots,n, \end{aligned} \tag{7}$$

which has a familiar regression model form. We model the $\boldsymbol{\epsilon}_{(i_1\, i_2)\, j}$ as independent and identically distributed multivariate Gaussian with a zero mean vector and covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma^2 & 0 & \phi & 0 \\ 0 & \tau^2 & 0 & \delta \\ \phi & 0 & \sigma^2 & 0 \\ 0 & \delta & 0 & \tau^2 \end{pmatrix}. \tag{8}$$

The main objective of this paper is to derive a model for the variances of Y_{P_n} and Y_{S_n}. While they are parameters in the multivariate model of $\mathbf{Z}$, σ^2 and τ^2 are not of themselves the variances of Y_{P_n} and Y_{S_n}. The correct variance equation for these variables is derived in Section 4.2, Equation 9. Equation 7 makes clear the fact that for each i_1, i_2 combination, our multivariate model is essentially four univariate regression models that are probabilistically linked together with the multivariate error term $\boldsymbol{\epsilon}_{(i_1\, i_2)\, j}$.

The error covariance matrix Equation 8 has a special structure. As discussed in Section 4.2, we model the marginal covariance structure of the two analysts as equal (the 2×2 block covariance matrix forming the diagonal) with the assumption that an analyst's picking errors for P_n and S_n are not correlated. The off-diagonal block covariance matrix models the conjecture that the two analysts' phase pick errors for P_n (S_n) may be correlated, but an analyst's pick error for P_n is not correlated with another analysts' pick error for S_n . The thought is that the analysts may follow a common analysis paradigm (model factor $\mathbf{T}$) that would introduce a positive correlation between their phase pick errors for the same phase. These features are captured in the structure of the covariance matrix Equation 8 and ultimately in Equation 9.

Denote the total number of vector responses as N. Because each vector response has 4 observations, there are a total of $4N$ data points. Referring to the cell-means formulation (Equation 7), there are $4ps$ mean parameters, 2 covariate parameters, 4 covariance matrix parameters and 1 weight function parameter for a total of $4ps + 2 + 4 + 1$ parameters. To ensure a sufficient number of data vectors N for parameter estimation and data analysis, we must have $4N - (4ps + 3 + 4) > 1$, which is minimally satisfied if $N > ps + 2$. Ideally, we want $N \gg ps + 2$. Each data vector $\mathbf{Y}_{i_1\, i_2\, j}$ is realized from the phase time picks of two analysts, so we need a total of $2N$ analysts to perform the experiment.

Fewer analysts will require special attention because the experiment will be unbalanced (unequal number of analysts in each source by path combination). There might be a perception that the experiment we have proposed can be accomplished with the repeated use of a small number of analysts. As noted in Section 3, this experiment would require multiple measurements over time from each analyst, and would introduce a learning effect that could not be removed. We argue that this learning effect would result in a very poor estimate of phase pick variability and would make inferential statistical analysis very difficult. If this learning effect could be removed, a well established, repeated measures design could be applied; however the learning effect is problematic. From a statistical perspective, analysis results from an unbalanced experiment are much more defensible than an experiment that introduces dependence (temporal correlation) into the observations.

To conduct the experiment, we randomly assign two different analysts, with similar experience and training, to each seismic event and collect the analysts' picks of the phase arrival times. Note that each analyst will only be required to identify two phase arrival times. Thus, the experiment requires a nominal number of analysts but very little time from each analyst. The optimal experimental procedure involves a double blind implementation; that is, waveforms are given to an analyst in a operational setting where that analyst is not aware that his/her phase pick is a data point in an experiment, and the operational system/process is also unaware that an event is part of an experiment. We recognize that such an optimal experiment may not be feasible; however, we recommend that this is the standard that should be sought. This experimental design is illustrated in Figure 1.

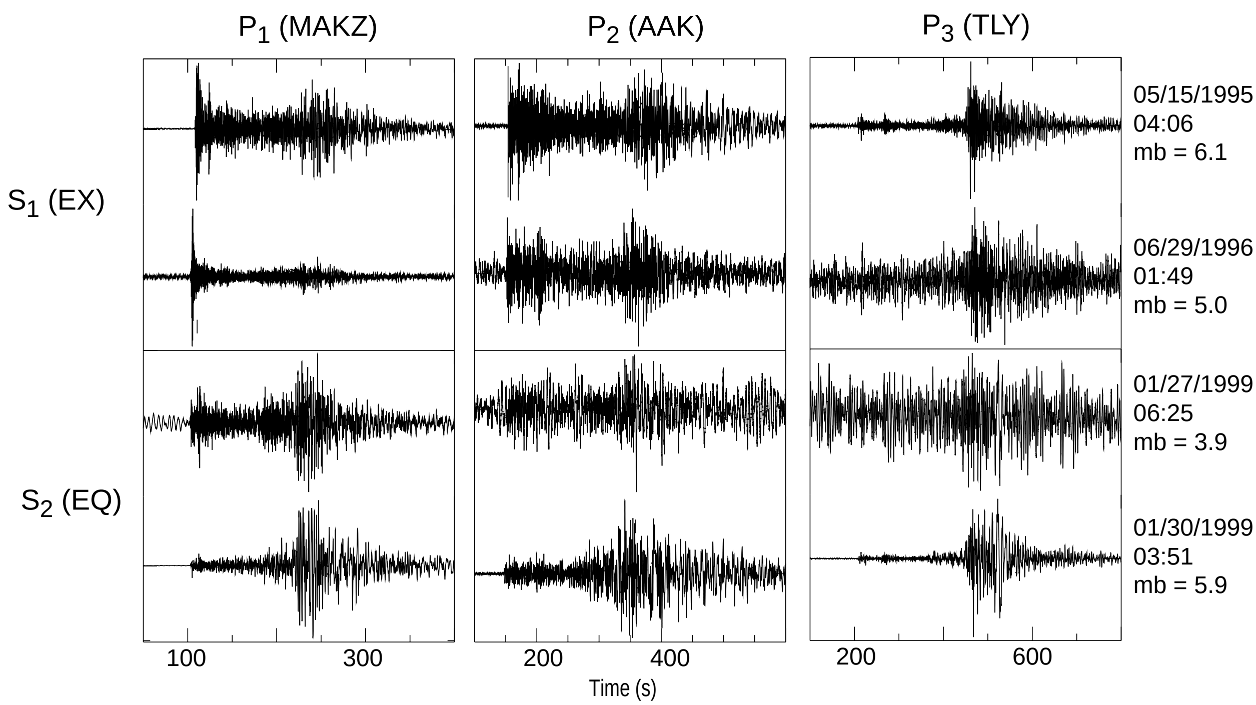

Figure 1: Experimental design with two event sources ($\mathbf{S}_{i_1}$), three wave paths ($\mathbf{P}_{i_2}$) and two distinct events in each cell. Two different randomly selected seismic analysts examine each waveform. Each analyst makes arrival time picks for the P_n and S_n phases. Event magnitude, and epicentral distance (source to seismometer) are integrated into the analysis as covariates.

4.1 Model Relevance

Let us compare our model (Equations 6 and 7) with the current operational measurement error model Equation 1. Equation 7 models dependence on SNR with the weight function Equation 3 and Equation 4, magnitude dependence with $\alpha \cdot mb$, epicentral distance dependence with $\beta \cdot \Delta$, source and path dependence with $\boldsymbol{\mu}_{i_1 i_2}$, and a true random component $\boldsymbol{\epsilon}$ reflecting error not accounted for with model terms. Equation 1 has a term for SNR dependence but is not adjusted for magnitude, epicentral distance, and path dependence. We believe that it also has an implicit term for random error — the measurement error cannot drop below a minimum value regardless of SNR, which essentially accounts for the existence of true random error. What of the missing terms in Equation 1? Perhaps these terms are not significant, but the reason for their omission in Equation 1 is more likely due to the fact that Equation 1 is used to assign measurement error to arrival time picks prior to event formation. At this stage neither the location nor the magnitude of the event is known, so the terms for magnitude dependence, distance dependence, and path dependence cannot possibly be used. Hence, Equation 1 is probably of the correct basic form, though it could be improved by allowing regionally dependent terms and by using a more sophisticated model for random error which would include the off-diagonal covariance terms. This model has utility even if the terms in Equation 6 are significant. Equation 6 (or 7) can be used to improve measurement error estimates as soon as an initial event location and magnitude have been determined. The net result of this is to give an improved estimate of the error ellipse associated with the location. Thus, we propose that operational systems should include at least two models for measurement error: phase pick error as a function of SNR alone (Equation 1) used prior to event location, and a more sophisticated model used after the event has been located (Equation 6). We discuss these ideas further in the next section.

4.2 Discussion

Assuming that a sufficient number of analysts is available, that a proper data set can be gathered and Equation 7 adequately models the analysts' phase arrival picks, how do we use Equation 7 to derive measurement error ($Cov(\mathbf{Y})$)? The answer to this question begins with the observation that, for known $\boldsymbol{\mu}$, α, β, and γ

$$Cov(\mathbf{Y}) = w(x,\gamma)^2 \; Cov(\mathbf{Z}) = w(x,\gamma)^2 \; \boldsymbol{\Sigma}. \tag{9}$$

Note that diagonal elements of Equation 9 are precisely Equation 1 when σ_0 is replaced with a variance component from $\boldsymbol{\Sigma}$. Therefore, the primary objective of our proposed model and analysis is to obtain an accurate estimate of the parameters in Equation 9 ($\boldsymbol{\Sigma}$ and γ). All other parameters in Equation 6 (or 7) properly account for systematic data structure and are necessary to obtain the most accurate estimates of $\boldsymbol{\Sigma}$ and γ. Thus, we use Equation 9 to obtain a phase pick error model that is conditional on the estimated values of $\boldsymbol{\mu}$, α, β, and γ.

In applying Equation 6 (or 7) to regional data, the covariate terms can be easily calculated given the event magnitude and distance to a station. We can also measure SNR at a station. Some logic difficulty arises in the use of the terms **S**, **P** and **SP** (or $\boldsymbol{\mu}_{i_1 i_2}$). These terms model a specific source type/wave path combination (e.g., an earthquake in central Italy recorded by a particular station) and therefore have values for these terms only for combinations that were included in our regression. Thus, we should have possibly three versions of Equation 9;

1. parameters $\mathbf{\Sigma}$ and γ estimated with the terms **S**, **P** and **SP** and all covariates removed from Equation 6

2. parameters $\mathbf{\Sigma}$ and γ estimated with the terms **S** and **SP** removed from Equation 6

3. parameters $\mathbf{\Sigma}$ and γ estimated with Equation 6 .

Version 1 could be used in initial seismic processing (event association). Version 2 could be used after an event has been formed (known path) but not identified (unknown source). Version 3 could be used after an event has been identified as an explosion and a more focused location confidence ellipse is necessary for on-site inspection activities. As a final note, it may be determined that the terms **S** and **SP** are not statistically significant and as a consequence versions 2 and 3 would be statistically equivalent.

5 Data Analysis

In this section we briefly outline data analysis options associated with Equation 6 (or 7). We refer to appropriate references for mathematical and statistical details. As stated in section 4.2, the primary objective of our model and analysis is to obtain an accurate estimate of the parameters in Equation 9. We propose maximum likelihood estimates (MLE) for these parameters and, in particular, iteratively re-weight maximum likelihood (least squares) estimation (see [Stuart et al., 1999]). An iterative technique is necessary because the weight Equation 3 is a function of the parameter γ. With regional data and associated SNR and covariate data we make an initial guess for γ and then weight the data (Equation 4). We then obtain standard MLE's for the parameters in Equation 6 (or 7) with the added constraint that the estimates of σ, ϕ, τ and δ (Equation 8) give a positive definite covariance matrix $\mathbf{\Sigma}$. The iterative procedure continues with a MLE update of γ and then the model parameters subject to the positive definite constraint for $\mathbf{\Sigma}$. This iterative procedure continues until the parameters numerically converge. The positive definite constraint for $\mathbf{\Sigma}$ can be mathematically summarized as $\sigma^2 > |\phi|$ and $\tau^2 > |\delta|$. From another perspective, these constraints ensure that the correlations formed from the elements of $\mathbf{\Sigma}$ are bounded between -1 and 1.

We can select a parsimonious model by determining if model terms are statistically significant. Likelihood ratio techniques can be used to perform these tests. The statistical and theoretical details of these analyses are detailed in [Anderson, 1984], [Mardia et al., 1995], [Press, 1982], [Rencher, 1995] and [Rencher, 1998]. Through these tests we may determine if the terms **S**, **P** and **SP** are necessary to adequately describe the data. We may also make this determination for the model covariates. We can study the adequacy of a fitted model by an analysis of agreement with model assumptions ([Rencher, 1995]). An analysis of model assumptions can also guide, if necessary, the construction of a more sophisticated version of Equation 3. Fitted model residuals can reveal model inadequacy (departures from model assumptions) as a result of emergent phases and other effects. Emergent phases introduce uncertainty in an analyst's ability to pick a phase arrival. For example, the arrival of the

P_n and P_g phases may be difficult to distinguish. This can result in violations of the model assumptions.

6 Summary and Future Work

We have embedded a general form of the IDC/USNDC phase pick measurement error model (Equation 1) into a multivariate statistical model (Equation 6). Our model can be used to optimally calibrate this picking error model to regional data. Our proposed model can also be used to study the statistical relevance of regional seismic factors that may influence analyst phase pick variability. We can assess the correlation that may be present between analysts' phase picks as a result of a common paradigm that analysts may use. Finally, we can use our statistical model to guide the construction of a more sophisticated version of Equation 1 and optimally calibrate this improved model to regional applications. This report proposes an analysis model and experimental design, and solicits expert technical feedback to improve our analysis strategy. We plan to apply the analysis outlined in this paper, with suggested improvements, to appropriate regional data and report on our findings in a follow-on paper.

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